DYNAMIC EPISTEMIC LOGIC MEETS PROBABILITY

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1 Logical dynamics, rational agency and intelligent interaction

From proof to information flow Inference patterns: $A \vee B, \neg A \Rightarrow B$, valid or invalid.

Agency involves a much broader range of correct information processing:

Restaurant: how to figure out who has which dish? *Inference, questions*.

Card games: planning moves using theory of mind. *Mutual knowledge*.

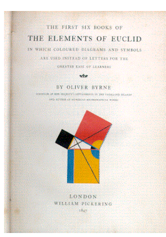
Skype Exam: secret voting on a public channel. *New social procedures*.

To be rational is to reason intelligently Logic of all basic informational processes:

"Zhi: Wen, Shuo, Qin" 知 问 说 亲 (communication, inference, observation)

To be rational is to act intelligently Add goals, preferences, decisions, actions.

To be rational is to interact intelligently Argumentation, communication, games.

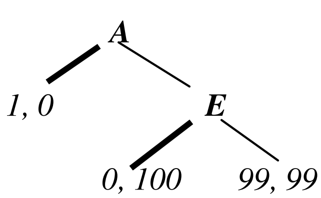




2 A new program for logic

Stage One: charting agents' informational abilities Knowledge update, belief revision (learning), inference dynamics, 'issue management' (questions, agenda). No consensus!

Stage Two: 'social dynamics': from single steps to long-term interaction Temporal processes, groups, interaction. Methods from logic, computer science, and *game theory*. Fine-structure of 'solution': what is involved in explaining/predicting behaviour?

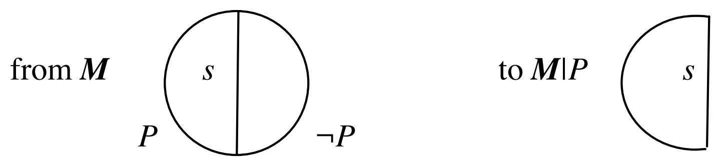


New mix of philosophical and computational logic – but classical mathematical tools.

3 Dynamifying epistemic logic to public announcement logic

Static base logic Language $p \mid \neg\phi \mid \phi \vee \psi \mid K_i\phi \mid C_G\phi$, models $\boldsymbol{M} = (W, \{\sim_i \mid i \in G\}, V)$, with worlds W, accessibility relations $\sim_i$, and valuation V. Truth conditions ('knowledge as semantic information'): $\boldsymbol{M}, s \models K_i\phi$ iff for all t with $s \sim_i t$: $\boldsymbol{M}, t \models \phi$, and $\boldsymbol{M}, s \models C_G\phi$ iff for all t reachable from s by some finite sequence of $\sim_i$ steps ($i \in G$): $\boldsymbol{M}, t \models \phi$.

ILLC-style dynamic logics describe key update steps in observation and communication. Pilot system: *PAL*. Hard information update: learning P eliminates worlds with P false:



Language extension: $\boldsymbol{M}, s \models [!P]\,\phi$ iff *if* $\boldsymbol{M}, s \models P$, *then* $\boldsymbol{M}|P, s \models \phi$

Theorem PAL axiomatized completely by epistemic logic plus *recursion axioms*:

$[!P]q$	$\leftrightarrow$	$P \rightarrow q$	*for atomic facts* q
$[!P]\neg\phi$	$\leftrightarrow$	$P \rightarrow \neg[!P]\phi$	
$[!P]\phi\wedge\psi$	$\leftrightarrow$	$[!P]\phi \wedge [!P]\psi$	
$[!P]K_i\phi$	$\leftrightarrow$	$P \rightarrow K_i(P \rightarrow [!P]\phi)$	*key recursion axiom*

Aside on 'schematic validities': $[!P][!Q]\phi \leftrightarrow [!(P \wedge [!P]Q)]\phi$

Methodology Add dynamic superstructure to static logic. *Compositional* analysis post-conditions. Requires *pre-encoding* in static language. E.g., $[!P]C_G\phi$ needs new notion: 'conditional common knowledge' with recursion axiom $[!P]C_G{}^{\psi}\,\phi \leftrightarrow C^{P \wedge [!P]\psi}\,[!P]\phi$

Hunt for right recursion axioms: private information, belief revision, questions. Describe information flow under many triggering events. Similar methods for preference or goals.

4 Product update: general events with partial observation

Social information flow involves variety. *Email*: epistemic-dynamic function of *cc*, *bcc*. *Games* designed to manipulate information flow (*Cluedo*). Partial observation of events.

Event models A $= (E, \{\sim_i \mid i \in G\}, \{PRE_e \mid e \in E\})$. Relevant events, relations $\sim_i$ encode agents' semantic range. I check my card: you cannot tell 'my seeing red' from 'my seeing black' (or more abstract invisible aspects). Events e have *preconditions* PRE_e for their execution: my having a red card, not knowing answer to my question, etc.

Update from epistemic $(\boldsymbol{M}, s)$ and event model $(\boldsymbol{E}, e)$ to ***product model*** $(\boldsymbol{M}x\boldsymbol{E}, (s, e))$:

Domain $\{(s, e) \mid s$ a world in $\boldsymbol{M}$, e an event in $\boldsymbol{E}$, $(\boldsymbol{M}, s) \models PRE_e\}$,

Accessibility: $(s, e) \sim_i (t, f)$ iff *both* $s \sim_i t$ *and* $e \sim_i f$,

Valuation for atoms p at (s, e) is that at s in $\boldsymbol{M}$.

Product update deals with misleading actions as well as truthful ones, and with *belief* as well as knowledge. Epistemic models can even get *larger* as update proceeds (*bcc*)!

Dynamic-epistemic logic LEA: $p \mid \neg\phi \mid \phi\vee\psi \mid K_i\phi \mid C_G\phi \mid [\boldsymbol{E}, a]\phi$: $(\boldsymbol{E}, e)$ any event model with actual event e. Semantics: $\boldsymbol{M}, s \models [\boldsymbol{E}, e]\phi$ iff $\boldsymbol{M}\ \boldsymbol{x}\ \boldsymbol{E}, (s, e) \models \phi$.

Theorem *LEA* is effectively axiomatizable and decidable.

The key recursion axiom is the one extending that for public announcement:

$$[\boldsymbol{E}, e]K_i\phi \quad \leftrightarrow \quad PRE_e \rightarrow \wedge \{ K_i[\boldsymbol{E}, f]\phi)) \mid f \sim_i e \text{ in } \boldsymbol{E}\}$$

Extensions to richer languages: factual change, common knowledge in subgroups etc.

5 Summary: main features of dynamic logics

* Combine information flow of many sorts: semantic, inferential, procedural, …
* Typical scenario is *social*: multi-agent interaction, groups as independent actors.
* Key to semantics is systematic new model construction as information flows.
* Background in epistemic *temporal logic*: an *ETL*-tree is representable as the result of iterated product update iff agents satisfy *Perfect Recall*, *Uniform No Miracles*.
* Issue 1: diversity of agent capacities, different complexities of their theories.
* Issue 2: *PAL* with *protocols*: role of 'procedural information'. Change in axioms: $<!P>q \leftrightarrow q$ now becomes $<!P>q \leftrightarrow q \wedge <!P>T$. Results by other methods.
* Dynamics of other powers: belief revision, inference.
* Logic interfaces with new areas: epistemology, informatics, game theory.
* Goal of the research still: mathematical formal systems. What is their status?

6 Probability makes sense in logic

* Enrich bare qualitative models
* Great 'aspect grinder'
* Compress past experience
* Smoothen computation
* Instrument for recognizing long-term emergent phenomena

7 Combining logic with probability

Probabilistic and logical update Conditionalizing probability is like *PAL* update. *PAL*'s multi-agent character orthogonal (?), but logic insists on update with arbitrary assertions.

Static epistemic probabilistic logic An *epistemic probability model* is a structure $\boldsymbol{M} = (W, \sim, P, V)$ with W a non-empty set of worlds, $\sim$ a set of equivalence relations $\sim_i$ on W for each agent i, P is a set of probability functions P_i assigning probability distributions for each agent i at each $w \in W$, and V a valuation assigning sets of states to proposition letters.

The *static epistemic-probabilistic language* is given by the following inductive syntax:

$\varphi ::= p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid K_i\varphi \mid P_i(\varphi) = q$, where q is a rational number, plus linear inequalities $\alpha_1 \bullet P_i(\varphi_1) + \ldots + \alpha_n \bullet P_i(\varphi_n) \geq \beta$ with $\alpha_1, \ldots, \alpha_n, \beta$ rational numbers.

This allows mixed formulas like $K_i P_j(\varphi) = k$, or $P_i(K_j\varphi) = k$. Key semantic clause:

$$\boldsymbol{M}, s \models P_i(\varphi) = q \qquad \text{iff} \qquad \sum_{s \text{ with } \boldsymbol{M}, s \models \varphi} P_i(s)(t) = q$$

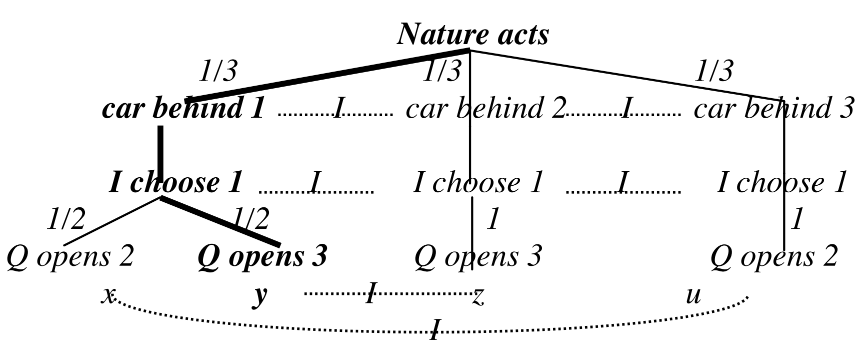
One can impose special conditions, such as $P_i(\varphi) = q \rightarrow K_i\, P_i(\varphi) = q$ (P-Introspection).

Dynamic proposals Kooi uses prior world probabilities in a model $\boldsymbol{M}$, and conditionalizes to get the new probabilities in $\boldsymbol{M}|A$ after public announcement $!A$. Recursion axiom:

$$[!A]\, P_i(\phi) = q \;\leftrightarrow\; P_i([!A]\phi \mid A) = q$$

To deal with Monty Hall, van Benthem used product update with event models encoding 'occurrence probabilities', creating probabilities for new events (s, e) with a rule

$$P^{\boldsymbol{M}x\boldsymbol{E}}_{i,\ (s, e)}((t, e)) \quad = \quad \frac{P^{\boldsymbol{M}}_{i,\ s}(t) \bullet P^{\boldsymbol{E}}_{t}(e)}{\Sigma_{u \sim I\, s \text{ in } \boldsymbol{M}}\, P_{i,\, s}(u) \bullet P_u(e)}$$



Bayes' Rule in update logic $P(\phi \mid A) = P(A \mid \phi) \bullet P(\phi) \;/\; P(A)$ holds in static base logic. But it can fail dynamically, with public announcements of epistemic statements A and ϕ.

8 Merging *DEL* with probabilistic update: dynamic logic with three factors

Three sources of probability (a) *Prior probabilities of worlds* in the current epistemic-probabilistic model ***M***, representing agents' current informational attitudes, (b) *occurrence probabilities for events* from the event model ***E*** encoding agents' views on what sort of process produces the new information, but also (c) *observation probability of events*, reflecting agents' uncertainty which event is actually being observed. *Examples.*

Probabilistic product update *Probabilistic event models* $\boldsymbol{E} = (E, \sim, \Phi, Pre, P)$ have (a) E is a non-empty finite set of events, (b) ~ is a set of equivalence relations $\sim_i$ on E for each agent i, (c) Φ is a set of pair-wise inconsistent sentences called 'pre-conditions' [new idea here], (d) *Pre* assigns to each pre-condition $\varphi \in \Phi$ a probability distribution over E (we write $Pre(\varphi, e)$, interpreting this as the probability that 'e occurs given φ'), and finally (e) for each i, the function P_i assigns to each event e a probability distribution over E.

Let ***M*** be an epistemic-probabilistic model and let ***E*** be a probabilistic event model. If s is a state in ***M***, write $Pre(s, e)$ for the value of $pre(\varphi, e)$ with φ the unique element of Φ that is satisfied at $\boldsymbol{M}, s$. If no such φ exists, set $pre(s, e) = 0$. Now, the *epistemic probabilistic product update model* $\boldsymbol{M} \times \boldsymbol{E} = (S', \sim', P', V')$ is defined by setting:

(a) $S' = \{ (s, e) \mid s \in S, e \in E \text{ and } pre(s, e) > 0\}$

(b) $(s, e) \sim_i (s', e')$ iff $s \sim_i s'$ and $e \sim_i e'$

(c) $P'_i((s, e), (s', e')) :=$

$$\frac{P_i(s)(s') \bullet Pre(s'; e') \bullet P_i(e)(e')}{\sum_{s'' \in S, e'' \in E} P_i(s)(s'') \bullet Pre(s'', e'') \bullet P_i(e)(e'')}$$

if the denominator > 0 and 0 otherwise.

(d) $V'((s, e)) = V(s)$

The new state space after the update consists of all pairs (s, e) where event e occurs with a positive probability in s (as specified by *Pre*). The crucial part are the new probability measures. The functions $P'_i (s, e)$ for (s', e') assign the arithmetical product of the prior probability for s', the probability that e' actually occurs in s', and the probability that i assigns to observing e'. To obtain a proper probability measure, we normalize. *Examples.*

Discussion and further developments (a) Theory of model construction, (b) Probabilistic bisimulation as measure of equality of models, (c) Modeling temporal protocols using 'intensional events' like observing agents of different types (also in learning theory).

Complete dynamic probabilistic logic A *dynamic-epistemic-probabilistic language*:

$\varphi ::= p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid K_i\varphi \mid P_i(\varphi) = q$, where q is a rational number, linear inequalities $\alpha_1 \bullet P_i(\varphi_1) + \ldots + \alpha_n \bullet P_i(\varphi_n) \geq \beta$ with $\alpha_1, \ldots, \alpha_n, \beta$ rational numbers, plus dynamic modality $[\boldsymbol{E}, e]\varphi$, where $\boldsymbol{E}$ is a probabilistic event model, and e an event from the domain of $\boldsymbol{E}$.

$$\boldsymbol{M}, s \models [\boldsymbol{E}, e]\varphi \text{ iff there is a } \psi \in \Phi \text{ with } \boldsymbol{M}, s \models \psi \text{ and } \boldsymbol{M} \boldsymbol{x} \boldsymbol{E}, (s, e) \models \varphi$$

Theorem The dynamic-epistemic probabilistic logic of update by probabilistic event models is completely axiomatizable, modulo some already given axiomatization of the base logic for the chosen class of static models.

Proof

$$\begin{aligned} P^{\boldsymbol{MxE}}(\psi) &= \sum_{(s', e') \text{ in } \boldsymbol{MxE}:\ \boldsymbol{MxE}, (s', e') \models \psi} P^{\boldsymbol{MxE}}(s', e') \\ &= \sum_{s' \in S,\ e' \in E:\ \boldsymbol{M}, s' \models \langle \boldsymbol{E}, e' \rangle \psi} P^{\boldsymbol{MxE}}(s', e') \\ &= \frac{\sum_{s' \in S,\ e' \in E:\ \boldsymbol{M}, s' \models \langle \boldsymbol{E}, e' \rangle \psi} P^{\boldsymbol{M}}(s') \bullet Pre(s', e') \bullet P^{\boldsymbol{E}}(e')}{\sum_{s'' \in S,\ e'' \in E} P^{\boldsymbol{M}}(s'') \bullet Pre(s'', e'') \bullet P^{\boldsymbol{E}}(e'')} \\ &= \frac{\sum_{\varphi \in \Phi,\ e' \in E} P^{\boldsymbol{M}}(\varphi \wedge \langle \boldsymbol{E}, e' \rangle \psi) \bullet k_{\varphi, e'}}{\sum_{\varphi \in \Phi,\ e'' \in E} P^{\boldsymbol{M}}(\varphi) \bullet k_{\varphi, e''}} \end{aligned}$$

where, for each φ and f, $k_{\varphi, f}$ is a constant, namely the value $Pre(\varphi, f) \bullet P^E(f)$.

This gives a recursion. Enumerate the finite set of preconditions $\boldsymbol{\Phi}$ and domain of $\boldsymbol{E}$ as $\varphi_0, \ldots, \varphi_n$ and $e_0, \ldots, e_m$. Then rewrite $\langle \boldsymbol{E}, e \rangle\ P(\psi) = r$, with '$P$' the probability after update, to an equivalent equation in which 'P' refers to probabilities in the prior model:

$$\frac{\sum_{1 \leq i \leq n,\ 1 \leq j \leq m} k_{\varphi i, ej} \bullet P(\varphi_i \wedge \langle \boldsymbol{E}, e_j \rangle \psi)}{\sum_{1 \leq i \leq n,\ 1 \leq j \leq m} k_{\varphi i, ej} \bullet P(\varphi_i)} = r$$

And the latter can be rewritten as a sum of terms:

$$\sum_{1 \leq i \leq n,\ 1 \leq j \leq m} k_{\varphi i, ej} \bullet P(\varphi_i \wedge \langle \boldsymbol{E}, e_j \rangle \psi) + \sum_{1 \leq i \leq n,\ 1 \leq j \leq m} -r \bullet k_{\varphi i, ej} \bullet P(\varphi_i) = 0$$

This fits in our language with linear inequalities – and we can also reduce the latter. ■

9 Further issues

Learning policies More flexibility: weigh the three factors differently, as in inductive logic. Extreme case: ‘over-ruling’. *Jeffrey Update* counts only observation probability.

Weighted Product Update Rule

$$P^{new}((s, e); (s', e')) := \frac{P(s)(s' \mid \varphi_{s'}) \bullet P(s)(s')^{\alpha} \bullet Pre(s', e')^{\beta} \bullet P(e)(e')^{\gamma}}{\sum_{s''\in S, e''\in E} P(s)(s'' \mid \varphi_{s''}) \bullet P(s)(s'')^{\alpha} \bullet Pre(s'', e'')^{\beta} \bullet P(e)(e'')^{\gamma}}$$

if the denominator > 0 – and 0, otherwise.

Zero probability and surprise What to do with cases that defy ‘accommodation’?

Plausibility versus probability Belief: should $(B\varphi \wedge B\psi) \rightarrow B(\varphi\wedge\psi)$ be valid? Belief revision via Priority update (Baltag & Smets): observation plausibility in ***E*** over-rules ***M***, as far as it goes, other plausibility in the product model ***MxE*** goes via prior plausibility. Different methodology: one update rule, but variety in complex inputs, viz. event models. Shift of learning rule into the structure of complex inputs: event models with rich signals. How does this compare with the system proposed here?

Logical analysis of probability How to deal with typical mixtures like *expected value?*

Practical uses of logical systems Difficulty. Probability rules as ‘hybrid calculation’.

References 1996, *Exploring Logical Dynamics*, CSLI, Stanford. 2009, *Dynamic Logics of Information and Interaction*, Cambridge UP. J. van Benthem, J. Gerbrandy & B. Kooi, 2009, ‘Dynamic Update with Probabilities’, to appear in *Studia Logica*.